B.Sc part-II (H) paper-III 14-07-2020

Differential Calculus. Tangents and normals.

Q.1. Prove that the eqn of the normal to the ~~the~~ curve $y = f(x)$ at the point (x, y) is $(X-x) + (Y-y)\frac{dy}{dx} = 0$ and that to the curve.

$f(x, y) = 0$ at (x, y) is $\frac{X-x}{f_x} = \frac{Y-y}{f_y}$

Soln. We know that normal to a curve at a point we mean the straight line which passes through that point and is at right angles to the tangent at the point.

The eqn of the tangent to the curve $y = f(x)$ at a point (x, y) is

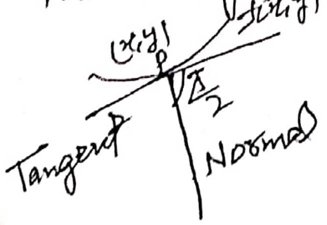
$$Y - y = \frac{dy}{dx}(X - x), \quad [\text{whose slope is } \frac{dy}{dx}]$$

Therefore, the slope of the normal $= -\frac{1}{\frac{dy}{dx}}$

Hence, the eqn. to the normal at the point (x, y) is

$$Y - y = \frac{1}{-\frac{dy}{dx}}(X - x) \Rightarrow (X - x) + (Y - y)\frac{dy}{dx} = 0$$

If the eqn of the curve be $f(x, y) = 0$, then eqn of the tangent is $(X - x) f_x + (Y - y) f_y = 0$ [where slope $= -\frac{f_x}{f_y}$]



$\therefore$ the slope of the normal $\frac{f_y}{f_x}$.

Hence, the eqn of the normal is

$$Y - y = \frac{f_y}{f_x}(X - x)$$

$$\Rightarrow \frac{X - x}{f_x} = \frac{Y - y}{f_y} \quad \text{Proved.}$$

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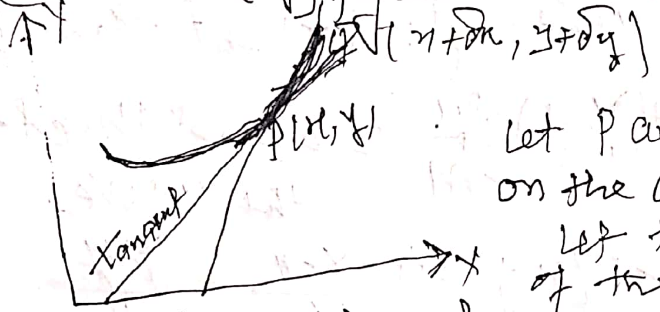
Q. #2. Prove that the eqn to the tangent to the curve $y = f(x)$ at (x, y) is $Y - y = \frac{dy}{dx}(X - x)$ and that to the curve $f(x, y) = 0$ is

$$(X - x) f_x + (Y - y) f_y = 0$$

Soln:-



Let P and Q be two points on the curve. Let the co-ordinates of the points P and Q be respectively (x, y) and $(x+\delta x, y+\delta y)$

Let the eqn to the curve be $y = f(x)$.

Then, $y + \delta y = f(x + \delta x)$, as $Q(x+\delta x, y+\delta y)$ is on the curve.

$$\therefore \frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}$$

Let (X, Y) be the current co-ordinates.

Joining the points $P(x, y)$ and $Q(x+\delta x, y+\delta y)$ is the eqn of st. line is

$$\frac{Y - y}{X - x} = \frac{(y + \delta y) - y}{(x + \delta x) - x} \Rightarrow \frac{Y - y}{X - x} = \frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}$$

When $Q \to P$. Then $\delta x \to 0$ and the straight line PQ will become the tangent to the curve at $P(x, y)$.

$$\therefore \frac{Y - y}{X - x} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$= f'(x) \; \frac{dy}{dx}$$

Hence, the eqn to the tangent to the curve $y = f(x)$ at $P(x, y) \Rightarrow Y - y = \frac{dy}{dx}(X - x) \longrightarrow \text{(I)}$

Now, if the eqn of curve be given in the form $f(x, y) = 0$.

$$\frac{dy}{dx} = -\frac{\partial f / \partial x}{\partial f / \partial y}$$

From (I), we get

$$Y - y = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}(X - x)$$

$$\Rightarrow (X - x)\frac{\partial f}{\partial x} + (Y - y)\frac{\partial f}{\partial y} = 0 \Rightarrow (X - x) f_x + (Y - y) f_y = 0$$

Which is the eqn of the tangent to the curve $f(x, y) = 0$ at $P(x, y)$.

Proved.

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